

**DHANALAKSHMI SRINIVASAN ENGINEERING COLLEGE****(AUTONOMOUS)**

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Accredited by NBA for AERO, BME, CSE, ECE, EEE, IT & MECH.

PE RAMBALUR-621212, TAMILNADU, INDIA.Website: www.dsengg.ac.in**COURSE PLAN (2025-2026 ODD SEMESTER)**

Course Code/Name	U20 ME701 / FINITE ELEMENT ANALYSIS			
Year/Section/Department	IV / B/Mechanical Engineering			
Credits Details	L: 3	T: 0	P: 0	C: 3
Total Contact Hours Required	45			

Syllabus:

UNIT I/ INTRODUCTION	No. of Periods : 9
Historical Background – Mathematical Modeling of field problems in Engineering – Governing Equations – Discrete and continuous models – Boundary, Initial and Eigen Value problems– Weighted Residual Methods – Variational Formulation of Boundary Value Problems – Ritz Technique – Basic concepts of the Finite Element Method.	
UNIT II/ ONE-DIMENSIONAL PROBLEMS	No. of Periods : 9
One Dimensional Second Order Equations – Discretization – Element types- Linear and Higher order Elements – Derivation of Shape functions and Stiffness matrices and force vectors- Assembly of Matrices - Solution of problems from solid mechanics and heat transfer. Longitudinal vibration frequencies and mode shapes. Fourth Order Beam Equation – Transverse deflections and Natural frequencies of beams.	
UNIT III/ TWO DIMENSIONAL SCALAR VARIABLE PROBLEMS	No. of Periods : 9
Second Order 2D Equations involving Scalar Variable Functions – Variational formulation – Finite Element formulation – Triangular elements – Shape functions and element matrices and vectors. Application to Field Problems - Thermal problems – Torsion of Non circular shafts – Quadrilateral elements – Higher Order Elements.	
UNIT IV/ TWO DIMENSIONAL VECTOR VARIABLE PROBLEMS	No. of Periods : 9
Equations of elasticity – Plane stress, plane strain and axisymmetric problems – Body forces and temperature effects – Stress calculations - Plate and shell elements.	
UNIT V/ ISOPARAMETRIC FORMULATION	No. of Periods : 9
Natural co-ordinate systems – Isoparametric elements – Shape functions for iso parametric elements– One and two dimensions – Serendipity elements – Numerical integration and application to plane stress problems - Matrix solution techniques – Solutions Techniques to Dynamic problems – Introduction to Analysis Software.	

Objective:

- To introduce the concepts of Mathematical Modeling of Engineering Problems.
- To appreciate the use of FEM to a range of Engineering Problems.

Text Book

T1: Reddy. J.N., "An Introduction to the Finite Element Method", 3rd Edition, Tata McGraw-Hill, 2006
T2: Seshu, P, "Text Book of Finite Element Analysis", Prentice-Hall of India Pvt. Ltd., New Delhi, 2007.

Reference Book

R1: Bhatti Asghar M, "Fundamental Finite Element Analysis and Applications", John Wiley & Sons, 2005 (Indian Reprint 2013)
R2: Chandrupatla & Belagundu, "Introduction to Finite Elements in Engineering", 3rd Edition, Prentice Hall College Div, 2011
R3: Logan, D.L., "A first course in Finite Element Method", Thomson Asia Pvt. Ltd., 2002
R4: Rao, S.S., "The Finite Element Method in Engineering", 3rd Edition, Butterworth Heinemann, 2010
R5: Robert D. Cook, David S. Malkus, Michael E. Plesha, Robert J. Witt, "Concepts and Applications of Finite Element Analysis", 4th Edition, Wiley Student Edition, 2002.

Website

W1: <https://www.sanfoundry.com/1000-finite-element-method-questions-answers/>
W2: <https://www.omicsonline.org/open-access/basic-concepts-of-finite-element-analysis-and-its-applications-in-dentistry-an-overview-2332-0702.1000156.php?aid=32322>
W3: <https://onlinelibrary.wiley.com/doi/abs/10.1002/nme.1620190603>

Online Mode of Study (if Any):

- ❖ <https://nptel.ac.in/courses/112104193/5>
- ❖ <https://nptel.ac.in/courses/112104193/14>
- ❖ <https://nptel.ac.in/courses/112104193/17>
- ❖ <https://nptel.ac.in/courses/112104193/20>
- ❖ <https://nptel.ac.in/courses/112104193/24>

U20ME701-Finite Element Analysis
(Important Questions)

Part-A

Unit I - INTRODUCTION

1) What is meant by finite element analysis?

Finite element method is a numerical method for solving problems of engineering mathematical physics. In the finite element method, instead of solving the problem for the entire body in one operation, we formulate the equations for each finite element and combine them to obtain the solution of the whole body.

2) What is meant by finite element?

A small unit having definite shape of geometry and nodes is called finite element.

3) State the methods of engineering analysis.

There are three methods of engineering analysis.

1. Experimental methods.
2. Analytical methods.
3. Numerical methods or approximate methods.

4) Give examples for the finite element.

1. One dimensional elements:
(a) Truss element (b) Bar, Beam element
2. Two dimensional elements
(a) Triangular element (b) Rectangular elements
3. Three dimensional elements
(a) Tetrahedral elements (b) Hexahedral elements.

5) What is meant by node or Joint?

Each kind of finite element has a specific structural shape and is interconnected with the adjacent elements by nodal points or nodes. At the nodes, degrees of freedom are located. The forces will act only at nodes and not at any other place in the element.

6) What do you mean by discretization?

Discretization is the basis of finite element method. The art of subdividing a structure into a convenient number of smaller components is known as discretization.

7) What are the types of boundary conditions?

There are two types of boundary conditions,
a) Primary boundary condition. b) Secondary boundary condition.

8) What are the three phases of finite element method.

The three phases are 1. Preprocessing 2. Analysis 3. Postprocessing

9) What is structural and non-structural problem?

Structural problem: In structural problems, displacement at each nodal point is obtained. By using these displacement solutions, stress and strain in each element can be calculated.

Non Structural problem: In non structural problem, temperatures or fluid pressure at each nodal point is obtained. By using these values, Properties such as heat flow, fluid flow, etc for each element can be calculated.

10) What are the methods are generally associated with the finite element analysis?

The following two methods are generally associated with the finite element analysis. They are 1. Force method. 2. Displacement or stiffness method

11) Name the variational methods.

1. Ritz method. 2. Rayleigh – Ritz method

12) Name the weighted residual methods.

1. Point collocation method.
2. subdomain collocation method.
3. Least square method
4. galerkin's method

13) What is meant by post processing?

Analysis and evaluation of the solution results is referred to as post processing. Post processor computer programs help the user to interpret the results by displaying them in graphical form.

14) What is Rayleigh ritz method?

Rayleigh ritz method is a integral approach method which is useful for solving complex structural problems, encountered in finite element analysis. This method is possible only if a suitable functional is available.

15) What is natural co-ordinates?

A natural co-ordinate system is used too defined any point inside the element by a set of dimensionless numbers, whose magnitude never exceeds unity. This system is very useful in assembling of stiffness matrices.

Unit II – One Dimensional Problems

1) What does assemblage mean?

The art of subdividing a structure into a convenient number of smaller components is known as discretization. These smaller components are then put together. The process of uniting the various elements together is called assemblage.

2) What is meant by DOF?

When the force or reaction acts at nodal point, node is subjected to deformation. The deformation includes displacement, rotations, and/or strains. These are collectively known as degrees of freedom (DOF).

3) What is aspect ratio?

Aspect ratio is defined as the ratio of the largest dimension of the element to the smallest dimension. In many cases, as the aspect ratio increases, the inaccuracy of the solution increases. The conclusion of many researches is that the aspect ratio should be close to unity as possible.

4) What is truss element?

The truss elements are the part of a truss structure linked together by point joints, which transmit only axial force to the element.

5) List the two advantages of post processing?

1. Required result can be obtained in graphical form.
2. Contour diagrams can be used to understand the solution easily and quickly.

6) State the assumptions are made while finding the forces in a truss.

The following assumptions are made while finding the forces in a truss.

1. All the members are pin jointed.
2. The truss is loaded only at the joints.
3. The self-weight of the members is neglected unless stated.

7) Write down the expression of shape function N and displacement u for one dimensional bar element.

For one dimensional bar element

Displacement function, $u = N_1 u_1 + N_2 u_2$

Where, shape function $N_1 = l-x / l$

shape function $N_2 = x/l$

8) During discretization, mention the places where it is necessary to place a node.

The following places are necessary to place a node during discretization process.

1. Concentrated load-acting point.
2. Cross section changing point
3. Different material inter junction point
4. Sudden change in load point.

9) Define frequency of vibration.

It is the number of cycles described in one second. Unit is Hz

10) Differentiate between global and local axes.

Local axes are established in an element. Since it is in the element level, they change with the change in orientation of the element. The direction differs from element to element.

Global axes are defined for the entire system. They are same in direction for all the elements even though the elements are differently oriented.

11) What are the types of loading acting on the structure?

There are three types of loading acting on the body. They are:

1. Body force (f)
2. Traction force (T)
3. Point load (P)

12) Define damping ratio.

It is defined as the ratio of actual damping coefficient to the critical damping coefficient.

13) What is meant by longitudinal vibrations?

When the particles of the shaft or disc moves parallel to the axis of the shaft, then the vibrations are known as longitudinal vibrations.

14) What is meant by transverse vibrations?

When the particles of the shaft or disc moves approximately perpendicular to the axis of the shaft, then the vibrations are known as transverse vibrations.

15) What is the classification of co-ordinates?

The co ordinates are generally classified as follows:

1. Global co-ordinates
2. Local co-ordinates
3. Natural co-ordinates

Unit III – Two Dimensional Scalar Variable Problems

1) Distinguish between potential energy function and potential energy functional.

If a system has finite number of degrees of freedom (q_1 q_2 and q_3)

then the potential energy is expressed as,

$G = f(q_1, q_2 \text{ and } q_3)$ It is known as function.

If a system has infinite degrees of freedom, then the potential energy is expressed as,

$G = H f.(x, y, dy/dx, d^2y/dx^2, \dots)dx$ It is known as functional.

2) What is the difference between static and dynamic analysis?

Static analysis: The solution of the problem does not vary with time is known as static analysis.

Example: Stress analysis on a beam.

Dynamic analysis: The solution of the problem varies with time is known as dynamic analysis.

Example: vibration analysis problems.

3) What are the basic steps involved in the finite element modeling.

1. Discretization of structure
2. Numbering of nodes.

4) What is discretization?

The art of subdividing a structure into a convenient number of smaller components is known as discretization.

5) Write down the general finite element equation.

General finite element equation is, $\{F\} = [K] \{u\}$

Where, $\{F\}$ - Force vector [column matrix]

$[k]$ - Stiffness matrix [row matrix]

$\{u\}$ - Degrees of freedom [column matrix]

6) State the assumptions in the theory of pure torsion.

1. The material of the shaft is homogeneous, perfectly elastic and obeys Hooke's law.
2. Twist is uniform along the length of the shaft.
3. The stress does not exceed the limit of proportionality.
4. Strain and deformation are small.

7) Define total potential energy.

The total potential energy G of an elastic body, is defined as the sum of total strain energy U and potential energy of the external forces, (W) .

Total potential energy, $G = \text{Strain energy } (U) + \text{Potential energy of the external forces } (W)$.

8) State the principle of minimum potential energy.

The principle of minimum potential energy states: Among all the displacement equations that satisfy internal compatibility and the boundary conditions, those that also satisfy the equations of equilibrium make the potential energy a minimum in a stable system.

9) What is the stationary property of total potential energy?

If a body is in equilibrium, its total potential energy G is stationary.

For stable equilibrium, $\delta^2 G > 0$, other wise G is minimum for stable equilibrium.

For neutral equilibrium, $\delta^2 G = 0$. In this case G is unchanging.

For unstable equilibrium, $\delta^2 G < 0$, other wise G is maximum.

10) State the principle of virtual work?

A body is in equilibrium if the internal virtual work equals the external virtual work for every kinematical admissible displacement field.

11) How do you define two-dimensional elements?

Two dimensional elements are defined by three or nodes in a two dimensional plane (ie x,y plane). The basic element useful for two dimensional analysis is the triangular element.

12) What is CST element?

Three-noded triangular element is known Constant Strain Triangle. it has six unknown displacement degrees of freedom ($u_1, v_1, u_2, v_2, u_3, v_3$). The element is called CST because it has a constant strain through it.

13) What is LST element?

Six noded triangular elements are known as linear strain triangle (LST), It has twelve unknown displacement degrees of freedom.

The displacement functions for the element are quadratic instead of linear as in the CST.

14) Define plane strain analysis

Plane strain is defined to be a state of strain in which the strain normal to the xy plane and the shear strains are assumed to be zero.

15) Write down the stiffness matrix equation for two-dimensional CST element.

Stiffness matrix, $[k] = [B]^T [D] [B] A t$

Where, $[B]$ = Strain displacement matrix

$[D]$ = Stress strain matrix

A = Area of the element

t = Thickness of the element

Unit IV – Two Dimensional Vector Variable Problems

1) What is axisymmetric element?

Many three dimensional problems in engineering exhibit symmetry about an axis of rotation. Such types of problems are solved by a special two-dimensional element called the axisymmetric element.

2) What are the conditions for a problem to axisymmetric?

1. The problem domain must be symmetric about the axis of revolution.
2. All boundary conditions must be symmetric about the axis of revolution.
3. All loading condition must be symmetric about the axis of revolution.

3) Give the stiffness matrix equation for an axisymmetric triangular element.

Stiffness matrix, $[k] = 2\pi r A [B]^T [D] [B]$

Where, co-ordinate $r = (r_1 + r_2 + r_3) / 3$

A = area of the triangular element.

4) What are the ways in which a three dimensional problem can be reduced to a two dimensional approach.

1. Plane stress: One dimension is too small when compared to other two dimensions.
Example: gear – thickness is small
2. Plane strain: one dimension is too large when compared to other two dimensions
Example : Long pipe [length is long compared to diameter]
3. Axisymmetric : geometry is symmetric about the axis.
Example: cooling tower

5) What are the types of shell element?

1. Flat shell element
2. Solid shell element
3. Curved shell element
4. Degenerated shell element

6) Give examples for essential (forced or geometric) and non-essential (natural) boundary conditions.

The geometric boundary conditions are displacement, slopes, etc. the natural boundary conditions are bending moment, shear force, etc.

7) What are h and p versions of finite element method?

h version and p versions are used to improve the accuracy of the finite element method.

In h versions, the order of polynomial approximation for all elements is kept constant and the number of elements is increased.

In p version, the number of elements is maintained constant and the order of polynomial approximation of element is increased.

8) Write the four basic sets of elasticity equations.

1. Strain-displacement relationship equations
2. Stress-Strain relationship equations
3. Equilibrium equations
4. Compatibility equations

9) Distinguish between plane stress and plane strain problems.

Plane stress is defined to be a state of stress in which the normal stress and shear stress directed perpendicular to the plane are assumed to be zero.

Plane strain is defined to be a state of strain in which the strain normal to the xy plane and the shear strains are assumed to be zero.

10) What is polynomial type of interpolation functions are mostly used in FEM?

The polynomial type of interpolation functions are mostly used due to the following reasons:

1. It is easy to formulate and computerize the finite element equations.
2. It is easy to perform differentiation or integration.
3. The accuracy of the results can be improved by increasing the order of the polynomial.

11) Define magnification factor.

The ratio of the maximum displacement of the forced vibration to the static deflection under the static force is known as magnification factor.

12) What are the types of Eigen value problems?

Determinant based methods, Transformation based methods and Vector iteration methods.

13) State the principle of superposition.

It states that for linear systems, the individual responses to several disturbances or driving functions can be superposed on each other to obtain the total response of the system.

14) Define shell element.

A shell element is curved surface, which by virtue of their shape can withstand both membrane and bending forces. A shell structure can take higher loads, membrane stresses and predominant, which is primarily caused due to in-plane forces.

15) Define plate element.

A plate is a planar structure with a very small thickness in comparison to the planar dimensions. The forces applied on a plate are perpendicular to the plane of the plate.

Unit V – Isoparametric Formulation

- 1) What is the purpose of isoparametric elements?

It is difficult to represent the curved boundaries by straight edges finite elements.

A large number of finite elements may be used to obtain reasonable resemblance between original body and assemblage. In order to overcome this drawback, isoparametric elements are used i.e. for problems involving curved boundaries; a family of elements known as “isoparametric elements” is used

- 2) Write down the shape function for 4 noded rectangular elements using natural co-ordinate system.

Shape functions: $N_1 = \frac{1}{4} (1 - \xi) (1 - \eta)$
 $N_2 = \frac{1}{4} (1 + \xi) (1 - \eta)$
 $N_3 = \frac{1}{4} (1 + \xi) (1 + \eta)$
 $N_4 = \frac{1}{4} (1 - \xi) (1 + \eta)$

Where, ξ and η are natural co-ordinates.

- 3) Define super parametric element.

If the number nodes used for defining the geometry is more than number of nodes used for defining the displacements is known as super parametric element.

- 4) What is meant by sub parametric element?

If the number of nodes used for defining the geometry is less than number of nodes used for defining the displacements known as isoparametric element.

- 5) What is meant by iso parametric element?

If the number of nodes used for defining the geometry is same as number of nodes used for defining the displacements is know as isoparametric element.

- 6) Is beam element an isoparametric element?

Beam element is not an isoparametric element since the geometry and displacements are defined by different order interpolation functions.

- 7) What is the difference between natural co-ordinates and simple natural coordinate?

A natural co-ordinate is one whose value lies between zero and one.

Examples: $L_2 = x/l$; $L_1 = (1-x/l)$

Area co-ordinates : $L_1 = A_1/A$; $L_2 = A_2/A$; $L_3 = A_3/A$

A simple natural co ordinates is one whose value lies between -1 to +1

- 8) Give examples for essential (forced or geometric) and non-essential (natural) boundary conditions.

The geometric boundary conditions are displacement, slopes, etc.

The natural boundary conditions are bending moment, shear force, etc.

9) What are the difference between 2 Dimensional scalar variable and vector variable elements?
 Two dimensional scalar variable elements have only one direction independent variable per node. Two dimensional triangular element stiffness matrix size is 3×3 .
 Two dimensional vector variable elements have direction dependent variable at each node.
 Two dimensional triangular element stiffness matrix size is 6×6 .

10) What are the types of non-linearity?
 1. Non-linearity in material behavior from point to point.
 2. Non-linearity in loading-deformation relation.
 3. Geometric non-linearity.
 4. Change in boundary condition for different loading.

11) Name the four FEA softwares?
 1. ANSYS 2. NASTRAN 3. COSMOS 4. NISA

12) Define body force (f).
 A body force is distributed force acting on every elemental volume of the body.
 Unit: Force per unit volume.
 Example: Self-weight due to gravity

13) Define traction force (T)
 Traction force is defined as a distributed force acting on the surface of the body
 Unit: force per unit area.
 Examples: Frictional resistance, viscous drag, surface shear etc.

14) What is point load (P)
 Point load is force acting at a particular point, which causes displacement.

15) Define shape function.
 In finite element method, field variables within an element are generally expressed by the following approximate relation, $u = N_1(x,y) u_1 + N_2(x,y) u_2 + N_3(x,y) u_3 + N_4(x,y) u_4$

Unit-1

1) A simply supported beam subjected to uniformly distributed load over entire span and it is Subjected to a point load at the centre of the span. Calculate the bending moment and Deflection at midspan using Rayleigh-Ritz method.

2) Find the deflection at the centre of a simply supported beam of span length l subjected to Uniformly distributed load throughout its length as shown in fig.1. using weighted residual method in four types.

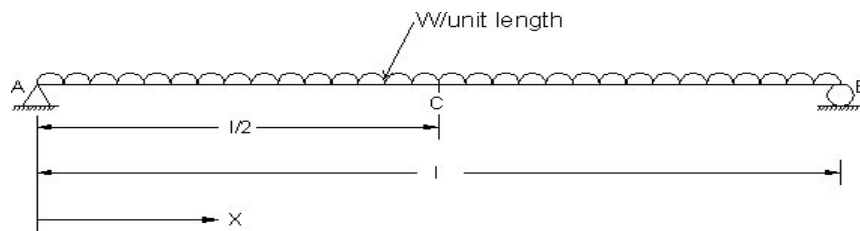


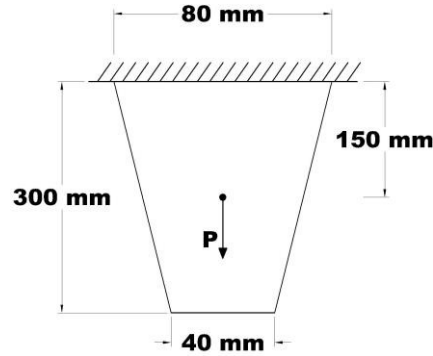
Fig. 1

- 3) A simply supported beam subjected to uniformly distributed load over entire span.
Determine the bending moment and deflection at midspan using Rayleigh-Ritz method.
- 4) Write a short note on Galerkins method.
- 5) Consider the differential equation, $d^2y/dx^2 + 400x^2 = 0$, for $0 < x < 10$, subject to boundary conditions are, $y(0) = 0$ and $y(10) = 0$, The functions corresponding to this problem, to be extremized is given by,

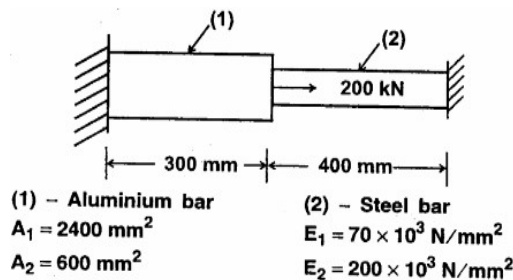
$$I = \int_0^{10} [-0.5 (dy/dx)^2 + 400x^2y] dx$$
 Find the solution of the problem using Raleigh Ritz method by considering a two term solution
 as $y(x) = c_1x(1-x) + c_2x^2(1-x)$
- 6) Explain the general steps in FEA with the help of a flowchart?
- 7) Describe about finite element analysis and write the advantages, disadvantages, application of FEA.
- 8) The following differential equation is available for a physical phenomenon.
 $d^2y/dx^2 + 50 = 0$, $0 < x < 10$, The trial function is, $y = ax(10-x)$
 The boundary conditions are $y(0) = 0$ and $y(10) = 0$
 Find the value of the parameter 'a' by (i) Point collocation method
 (ii) Sub-domain collocation method (iii) Least squares method (iv) Galerkin's method.
- 9) Describe the types of weighted residual methods.

Unit-II

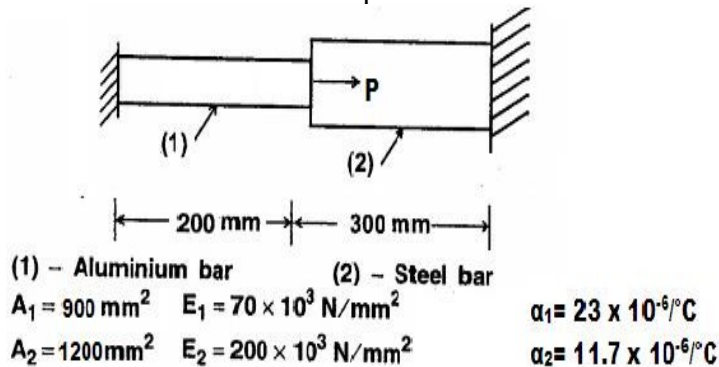
- 1) Derive the shape function and stiffness matrix for one-dimensional bar element.
- 2) For a tapered plate of uniform thickness $t = 10$ mm as shown in fig, find the displacements at the nodes by forming into two element model. The bar has mass density $\rho = 7800$ kg/m³, Young's modulus, $E = 2 \times 10^5$ MN/m². In addition to self weight, the plate is subjected to a point load $p = 10$ kN at its centre. Also determine the reaction force at the support.



3). A stepped bar is subjected to an axial load of 200 kN at the place of change of cross section and material as shown in figure . Find (a) The nodal displacements and (b) the induced stresses in each material



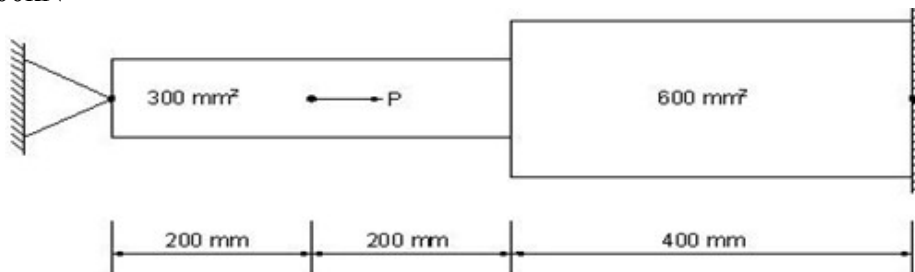
4). An axial load $P=300 \times 10^3 \text{ N}$ is applied at 20°C to the rod as shown in fig. The temperature is then raised to 60°C . Determine the nodal displacements.



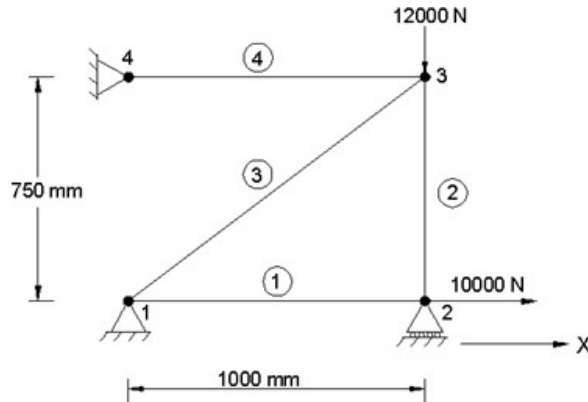
5). Determine the eigen values and natural frequencies of a system whose stiffness and mass matrices are given below.

$$[K] = \frac{2AE}{L} \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}, \quad [M] = \frac{\rho AL}{12} \begin{bmatrix} 6 & 1 \\ 1 & 2 \end{bmatrix}$$

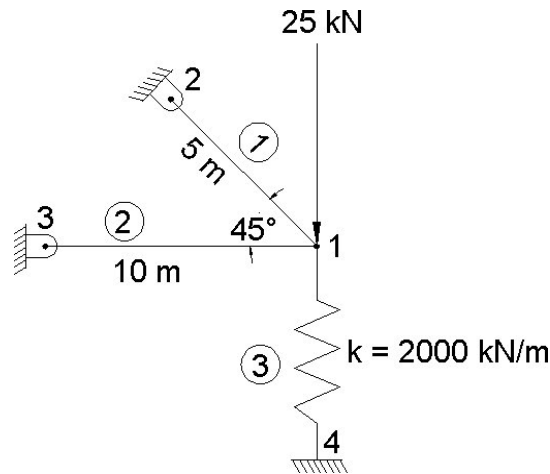
- 6). Determine the natural frequencies and mode shapes of transverse vibration for a beam fixed at both ends. The beam may be modeled by two elements, each of length L and cross-sectional area A . Consider lumped mass matrix approach.
- 7). A furnace wall is made up of three layers, inside layer with thermal conductivity 8.5 W/mK , the middle layer with conductivity 0.25 W/mK , the outer layer with conductivity 0.08 W/mK . The respective thicknesses of the inner, middle and outer layer are 25 cm , 5 cm and 3 cm respectively.
The inside temperature of the wall is $600 \text{ }^\circ\text{C}$ and outside of the wall is exposed to atmospheric air at $30 \text{ }^\circ\text{C}$ with heat transfer coefficient of $45 \text{ W/m}^2\text{K}$. Determine the nodal temperatures.
- 8). A steel rod of radius $r=1 \text{ cm}$, length $L=5 \text{ cm}$ and thermal conductivity $k=70 \text{ W/cm}^\circ\text{C}$ is exposed at one end to a constant temperature of 140°C . The atmospheric air of temperature 40°C with a convection coefficient of $10 \text{ W/cm}^2\text{ }^\circ\text{C}$. Determine the temperature distribution. The region can be discretized into 2 elements and 3 nodes.
- 9). Why higher order elements are needed? Determine the shape functions of an eight noded Rectangular element.
- 10). Derive the stiffness matrix $[K]$ for the truss element.
- 11). Consider the bar as shown in fig. Calculate the following:
(i) Nodal displacements, (ii) Element stresses and (iii) Support reactions.
Take
 $E=2 \times 10^5 \text{ N/mm}^2$
 $P=400 \text{ kN}$



- 12) Consider a four bar truss as shown in fig. It is given that $E = 2 \times 10^5 \text{ N/mm}^2$ and $A_e = 625 \text{ mm}^2$ for all elements.
- i) Determine the element stiffness matrix for each element.
 - ii) Assemble the structural stiffness matrix K for the entire truss.
 - iii) Solve for the nodal displacements.

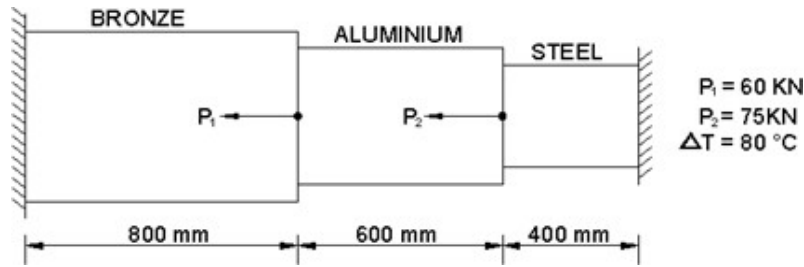


- 13) Consider a two-bar truss supported by a spring shown in Fig. Both bars have $E = 210 \text{ GPa}$ and $A = 5.0 \times 10^{-4} \text{ m}^2$. Bar one has a length of 5 m and bar two has a length of 10 m. The spring stiffness is $k = 2000 \text{ kN/m}$. Determine the horizontal and vertical displacements at the joint 1 and stresses in each bar.



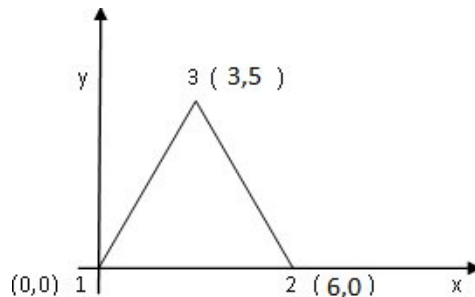
- 14). Derive the finite element equation for one dimensional heat conduction with free end convection
- 15). The structure shown in fig. is subjected to an increase in temperature of $80 \text{ }^\circ\text{C}$. Determine the
Displacements, stresses and support reactions. Assume the following data:

Bronze	Aluminium	Steel
$A = 2400 \text{ mm}^2$	1200 mm^2	600 mm^2
$E = 83 \text{ Gpa}$	70 Gpa	200 GPa
$\alpha = 18.9 \times 10^{-6}/^\circ\text{C}$	$23 \times 10^{-6}/^\circ\text{C}$	$11.7 \times 10^{-6}/^\circ\text{C}$



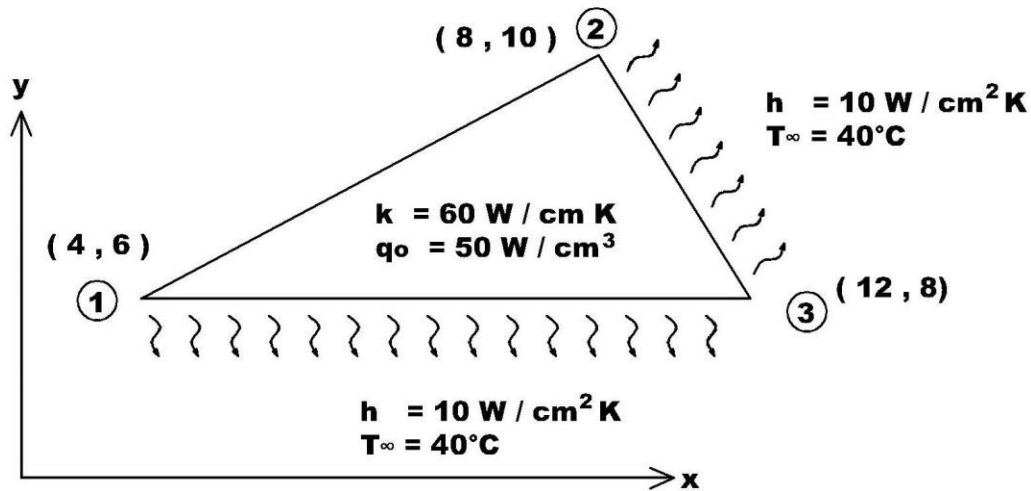
Unit-III

- 1) Derive the shape function for CST element, & give the name, formulae's in [K],[B]&[D] matrix.
- 2) The nodal coordinates of the triangular element are (2,2) (7,4) & (3,6). The nodal temperatures are 75°C, 90°C & 60°C. Calculate the shape functions and temperature at a point P(4,3)
- 3) The nodal coordinates of the triangular element are (1,2) (4,3) & (6,4) in mm. At the interior point P, the x coordinate is 3.5 and $N_1=0.4$, Calculate N_2 , N_3 and the y coordinate at point P.
- 4) Evaluate the element stiffness matrix and temperature force vector for the CST element at its three nodes are $(x_1,y_1)=(0,0)$, $(x_2,y_2)=(2,0)$ and $(x_3,y_3)=(1,3)$ in mm, under plane stress condition. The element experiences a 10°C increase in temperature. Assume $E=200$ GPa, $\nu=0.25$, $t=5$ mm. The coefficient of thermal expansion is $6 \times 10^{-6}/^\circ\text{C}$.
- 5) Evaluate the element strains and stresses for the triangular element its three nodes are (20, 30), (80, 30) and (50, 120) under plane stress condition. The coordinates are in mm. Assume the following values: $E=210$ GPa, $\nu=0.25$, $t=10$ mm. The nodal displacements are $u_1 = 2.0$ mm, $v_1 = 1.0$ mm, $u_2 = 0.5$ mm, $v_2 = 0.0$ mm, $u_3 = 3.0$ mm, $v_3 = 1.0$ mm
- 6) Evaluate the element stiffness matrix for the triangular element in following fig. The coordinates are in mm. under plane stress condition. Assume the following values: $E=200$ GPa, $\nu=0.25$, $t=1$ mm



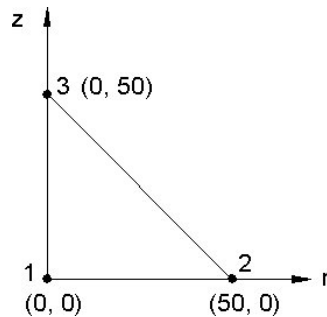
- 7) Derive the finite element equation for Torsional Bar element.

- 8) Compute the element matrix and vectors for the element shown in figure.
When the edges 2-3 and 3-1 experience convection heat loss.

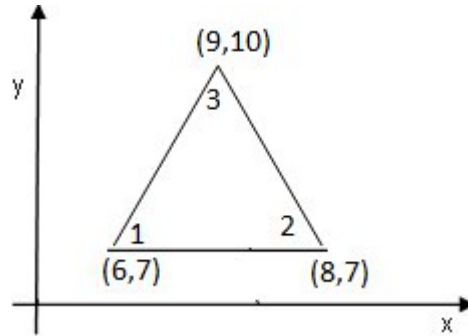


Unit-IV

- 1) Derive the shape functions for an axisymmetric triangular element.
- 2) Evaluate the element stiffness matrix for an axisymmetric triangular element shown in fig. Take following values: $E=210 \text{ GPa}$, $\mu=0.25$ (dimensions are in mm)



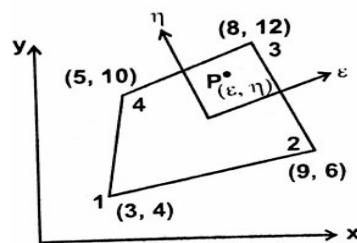
- 3) Calculate the element stiffness matrix and temperature force vector for an axisymmetric triangular element as shown in fig. The element experiences a 15°C increase in temperature. Take $E=200 \text{ GPa}$, $\mu=0.25$. The coefficient of thermal expansion is $10 \times 10^{-6}/^\circ\text{C}$. The coordinates are in mm.



- 4) To determine the element strains and stresses for an axisymmetric triangular element at its three nodes are $(r_1, z_1) = (3, 4)$, $(r_2, z_2) = (6, 5)$ and $(r_3, z_3) = (5, 8)$. The displacements are $u_1 = 0.002$, $w_1 = 0.001$, $u_2 = 0.001$, $w_2 = -0.004$, $u_3 = -0.003$ & $w_3 = 0.007$. (All dimensions are in cm). Assume $E = 210 \text{ GPa}$, $\mu = 0.25$
- 5) Derive an expression for the strain-displacement matrix for an axisymmetric triangular element.
- 6) The nodal coordinates for an axisymmetric triangular element at its three nodes are $(r_1, z_1) = (30, 10)$, $(r_2, z_2) = (50, 10)$, $(r_3, z_3) = (40, 60)$. Determine the strain displacement matrix for that element.
- 7) Explain the finite element analysis of shell element.
- 8) Explain the finite element analysis of plate element.
- 9) Describe about the serendipity element.

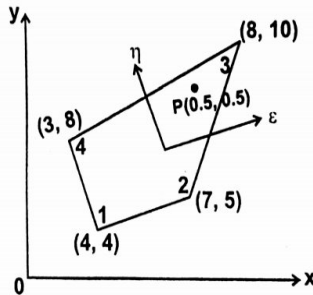
Unit-V

- 1) Derive the shape function and jacobian matrix for a 4 noded rectangular element.
- 2) Calculate the Cartesian coordinates of the point P which has local coordinates $\xi = 0.8$ and $\eta = 0.6$ as shown in figure



- 3) The nodes of the isoparametric quadrilateral element are $(3, 1)$, $(6, 1)$, $(8, 6)$ & $(2, 5)$. Determine the local coordinates of the interior point P, which has the Cartesian coordinates $(7, 4)$.

4) Evaluate the Jacobian matrix at the local coordinates $\xi=\eta= 0.5$ for the linear quadrilateral element with its global coordinates as shown in fig. Also evaluate the strain-displacement matrix



5) Consider the isoparametric quadrilateral element with nodes 1-4 at (0, 0) (2, 0) (2, 1) (0, 1) respectively.

Take $E= 2 \times 10^5 \text{ N/mm}^2$; $\nu= 0.25$; $u= [0, 0, 0.003, 0.004, 0.006, 0.004, 0, 0]^T$; $\sigma_{xx} = 0$, $\sigma_{yy} = 0$
 Determine the following, (Assume plane stress condition)

- (i) Jacobian matrix
- (ii) Strain displacement matrix
- (iii) Element strains
- (iv) Element stresses

6) Evaluate the integral $I= \int_{-1}^1 (a_1 + a_2x + a_3x^2 + a_4x^3)dx$ using the three point Gauss integration

7) Evaluate the following integral using two point Gaussian quadrature.

$$I = \int_{-1}^1 \int_{-1}^1 (1-2x)^2 (4-y-5) dx dy$$

8) Use Gaussian quadrature rule (n=2) to numerically integrate $\int_{-1}^1 \int_{-1}^1 xy dx dy$